

Photon-graviton conversion in a primordial magnetic field and the cosmic microwave background

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We reconsider the effects of photon-graviton conversion in a primordial magnetic field upon the cosmic microwave background radiation. We argue that plasma effects make the photon-graviton conversion process negligible.

I. INTRODUCTION

A photon that covers a distance L within a transverse, homogeneous magnetic field of strength B has a probability of converting into a graviton given by [1,2]

$$P \simeq 4\pi G B^2 L^2 \simeq 8 \times 10^{-50} \left(\frac{B}{\text{Gauss}} \right)^2 \left(\frac{L}{\text{cm}} \right)^2, \quad (1)$$

where G is Newton's constant.

It has recently been suggested [3,4] that a primordial magnetic field may imprint observable consequences upon the cosmic microwave background radiation through photon-graviton conversion. According to eq. (1), a primordial magnetic field of present value around 10^{-8} Gauss, if it already existed at the time of decoupling of matter and radiation and was homogeneous over a Hubble radius, would have induced a degree-scale anisotropy of the cosmic microwave background of about 10^{-5} , of the order of the observed value [5]. Although current bounds suggest that a cosmological magnetic field, if it exists, has present strength smaller than around 10^{-9} Gauss [6], photon-graviton conversion could in principle provide an independent method to constrain or eventually detect a primordial cosmological magnetic field.

In this article we wish to point out that plasma effects due to the Universe residual ionization make the photon-graviton oscillation length much shorter than the Hubble radius, and the probability of photon-graviton conversion is consequently much smaller than in the absence of free electrons. The effects of a primordial magnetic field of present value around 10^{-9} Gauss or smaller are consequently negligible.

II. PHOTON-GRAVITON CONVERSION PROBABILITY

The interaction between a gravitational and an electromagnetic field linearized in the small perturbation $h_{\mu\nu}$ around flat space-time is described, in General Relativity, by the term in the action

$$S_{int} = \frac{1}{2} \int h_{\mu\nu} T^{\mu\nu} d^4x \quad (2)$$

where $T_{\mu\nu}$ is the flat-space energy-momentum tensor of the electromagnetic field.

In an external, homogeneous magnetic field B , photons and gravitons can convert into each other conserving energy and linear momentum. The linearized interaction term between electromagnetic and gravitational plane-waves with the same wave-vector can be written as

$$S_{int} = B \sin \theta \int [h_+ E_\perp + h_\times E_\parallel] d^4x. \quad (3)$$

Here E_\parallel and E_\perp denote respectively the component of the electric field in the electromagnetic wave that is either parallel or perpendicular to the plane that contains the direction of propagation and the external homogeneous magnetic field, h_+ and h_\times describe two independent polarization modes of the gravitational wave, in the transverse-traceless gauge, and θ is the angle between the external magnetic field and the common direction of propagation of the electromagnetic and gravitational waves.

From eq. (3) the conversion probability between photons and gravitons is easily read off. Incoming photons with polarization either \parallel or \perp convert into gravitons with the same probability

$$P = 4\pi G B^2 L^2 \sin^2 \theta, \quad (4)$$

the only difference being the polarization of the resulting graviton.

We wish to point out that precisely because these two independent states of linear polarization have the same conversion probability, unpolarized electromagnetic radiation does not become linearly polarized due to photon-graviton conversion as it propagates through an homogeneous magnetic field. In this respect, photon-graviton conversion differs qualitatively from the conversion between photons and pseudoscalar particles [7,8]. In the latter case, only E_\parallel mixes with the pseudoscalar field.

Photon-pseudoscalar conversion in a cosmological magnetic field induces a small degree of linear polarization in the cosmic microwave background [9]. We conclude, however, and contrary to the claim in ref. [4], that photon-graviton conversion does not induce linear polarization in the cosmic microwave background.

In the presence of a free electron density n_e , photons propagate as if they had an effective mass equal to the plasma frequency $\omega_{pl}^2 = 4\pi\alpha n_e/m_e$, where m_e denotes the electron mass and $\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$ is the fine structure constant. We work in Heaviside-Lorentz natural units (in which $1\text{Gauss} = 1.95 \times 10^{-2}\text{eV}^2$). If the external magnetic field and the electron density are perfectly homogeneous, there are oscillations between the electromagnetic and gravitational plane waves, over an oscillation length given by [8]

$$\ell_{\text{osc}} = \frac{4\pi\omega}{\omega_{pl}^2} \quad (5)$$

where ω is the angular frequency of the electromagnetic wave. Indeed, the photon-graviton conversion probability, for either \parallel or \perp polarization, becomes [8]

$$P = \frac{4}{\pi} G B^2 \ell_{\text{osc}}^2 \sin^2 \left(\frac{\pi L}{\ell_{\text{osc}}} \right) \sin^2 \theta \quad (6)$$

Of course, if $L \ll \ell_{\text{osc}}$ this expression reduces to eq. (4), as if there were no free electrons. Otherwise, the conversion probability does not accumulate over distances larger than ℓ_{osc} .

The situation is different when there are processes, such as inhomogeneities in the electron-density, that affect the coherence of the photon-graviton oscillations. In this case a fraction f of the photons that mixed into gravitons within one oscillation does not oscillate back into photons. Adding the effect over $N = L/\ell_{\text{osc}}$ independent regions the photon-graviton conversion probability over a distance L becomes

$$P \simeq f G B^2 L \ell_{\text{osc}} \sin^2 \theta \quad (7)$$

The precise value of the factor f is model-dependent. See for instance ref. [10] for an estimate of these effects in the interstellar medium in our galaxy. For our purposes it will be enough to consider its largest possible value, $f \simeq 1$. We shall see that even in this most favourable case, photon-graviton conversion in a primordial magnetic field has negligible effects.

III. CMB ANISOTROPY INDUCED BY PHOTON-GRAVITON CONVERSION

Photon-graviton conversion in a cosmological magnetic field induces anisotropies in the CMB due to the angular dependence of the conversion probability [2–4]. Ignoring plasma effects, the conversion probability is frequency-independent, and thus preserves the black-body CMB

spectrum. Using eq. (4) we see that a cosmological magnetic field of present value $B(t_0)$ assumed homogeneous over a scale of order the present Hubble radius, H_0^{-1} , would induce (if plasma effects were negligible) a large angular scale anisotropy of order

$$\frac{\Delta T}{T} \simeq 5 \times 10^{-6} \left(\frac{B(t_0)}{1.3 \times 10^{-6} \text{Gauss}} \right)^2 \left(\frac{h}{0.5} \right)^{-2} \quad (8)$$

where $H_0 = 100 h \text{ km seg}^{-1} \text{Mpc}^{-1}$. The anisotropy induced at present times by a cosmological magnetic field of about 10^{-9} Gauss would thus be negligible, about six orders of magnitude smaller than the observed quadrupole CMB anisotropy [11], even in the absence of plasma effects.

A cosmological magnetic field of present value $B(t_0)$ is expected to have been larger in the past, by a factor $B(t) = B(t_0)a^2(t_0)/a^2(t)$, where a is the Robertson-Walker scale factor, due to flux conservation [6]. Photon-graviton conversion would thus have had larger effects in the past, if the magnetic field was always homogeneous over a Hubble radius, since the factor $(BH^{-1})^2$ scales with redshift as $1+z$ in a matter-dominated universe. Anisotropies induced before decoupling, however, are quickly erased by Thomson scattering during the period of tight coupling between photons, electrons and baryons. The largest effect would thus arise right around decoupling. The anisotropy induced around the time of decoupling of matter and radiation ($t = t_*$), on angular scales of order the size of the horizon at decoupling, which corresponds to about one degree on our sky is, neglecting plasma effects

$$\frac{\Delta T}{T} \approx 10^{-5} \left(\frac{B(t_*)}{0.04 \text{Gauss}} \right)^2 \left(\frac{h}{0.5} \right)^{-2} \left(\frac{1+z_*}{1100} \right)^{-3} \quad (9)$$

10^{-5} is the order of the observed anisotropy on angular scales of about one degree [5]. The present value of a primordial magnetic field which had a strength $B(t_*) \simeq 0.04 \text{ Gauss}$ at decoupling is $B(t_0) \simeq 3 \times 10^{-8} \text{ Gauss}$. We thus conclude, as in refs. [3,4], that if plasma effects were negligible the conversion between photons and gravitons in a primordial magnetic field around the time of decoupling of matter and radiation could have non-negligible effects upon the isotropy of the CMB.

Plasma effects, however, are not negligible. Even in the most favourable case, with $f \approx 1$ in eq. (7), the conversion probability drops precipitously. Consider the Universe right after decoupling of matter and radiation. The number density of free electrons is

$$n_e(t \approx t_*) = 0.15 \left(\frac{\Omega_b h^2}{0.01} \right) \left(\frac{1+z_*}{1100} \right)^3 \left(\frac{X}{10^{-3}} \right) \text{cm}^{-3} \quad (10)$$

where X is the fractional residual ionization and Ω_b is the baryon energy-density in units of the critical density. Notice that because the oscillation length depends on the photon frequency, so does the conversion probability. Photon-graviton conversion does not preserve the

black body spectrum of the CMB. We still write, for comparison purposes, the anisotropy in the CMB intensity induced by photon-graviton conversion in terms of an effective temperature anisotropy, at a given frequency. The anisotropy induced by a magnetic field homogeneous over a Hubble radius at decoupling would be at most, including plasma effects

$$\begin{aligned} \frac{\Delta T}{T} &\lesssim GB^2(t_*)H^{-1}(t_*)\ell_{\text{osc}}(t_*) \\ &\simeq 10^{-5} \left(\frac{B(t_*)}{14\text{Gauss}} \right)^2 \left(\frac{h}{0.5} \right)^{-1} \left(\frac{\nu(t_o)}{90\text{GHz}} \right) \end{aligned} \quad (11)$$

Here $\nu(t_o)$ is the present value of the CMB photons' frequency. A magnetic field of strength 14 Gauss at decoupling would have a strength of order 10^{-5} Gauss today. A realistic value, smaller than 10^{-9} Gauss today, would thus induce anisotropies through photon-graviton conversion at least eight orders of magnitude smaller than those observed.

We have already seen that the large angular scale anisotropy in the CMB induced at present times by a field of 10^{-9} Gauss would be negligible even in the absence of free electrons. A small electron-density would reduce the effect of photon-graviton conversion even further. The present value of the free electron density in the intergalactic medium is not known with certainty. The Gunn-Peterson limit on the abundance of neutral Hydrogen [12] however, suggests that most of the intergalactic material is ionized. A probably realistic figure for the present electron number density is thus $n_e \approx 10^{-7}\text{cm}^{-3}$. The anisotropy induced today on large angular scales by a cosmological magnetic field would thus be

$$\begin{aligned} \frac{\Delta T}{T} &\lesssim 5 \times 10^{-6} \left(\frac{B(t_o)}{0.006\text{Gauss}} \right)^2 \\ &\times \left(\frac{\nu}{90\text{GHz}} \right) \left(\frac{10^{-7}\text{cm}^{-3}}{n_e} \right) \left(\frac{h}{0.5} \right)^{-1} \end{aligned} \quad (12)$$

Clearly, the effect of a cosmological magnetic field of current value around 10^{-9} Gauss would be completely negligible.

IV. CONCLUSIONS

Photon-graviton conversion induced by a cosmological magnetic field of present strength 10^{-9} Gauss or smaller has negligible effects upon the isotropy of the cosmic microwave background. The effect would have been much larger in the absence of free electrons. Plasma effects, however, make the characteristic length for photon-graviton oscillations much smaller than the Hubble radius, preventing the conversion probability to grow quadratically with distance over such large scales.

We have also seen that photon-graviton conversion does not induce linear polarization upon the cosmic microwave background, contrary to the case of photon-pseudoscalar conversion [9,10].

The probability of photon-graviton conversion in a magnetic field in the presence of free electrons depends on the photon frequency. In principle, one could also attempt to detect the effects of a primordial magnetic field through departures from the black body spectrum in the CMB. Since thermalization processes are effective only at redshifts larger than about $z \approx 10^6$, one could test in this way for the presence of a primordial magnetic fields at very early times. One can show, however, that the departure from the black body spectrum is also negligibly small. For a present field of 10^{-9} Gauss at CMB frequencies of order 10^3 GHz, the fractional departure from a black body spectrum is at most of order 10^{-12} , induced right after decoupling. At earlier times, with matter fully ionized, the large free electron density makes the effect even smaller, of order 10^{-16} at the time of matter-radiation equality, $z_{\text{eq}} \approx 10^4$. At higher redshifts the factor $B^2 H^{-1} \ell_{\text{osc}}$ remains constant.

We should mention that a primordial magnetic field may still have significant effects upon the isotropy of the cosmic microwave background by driving an anisotropic expansion of the Universe [2]. Its direct effect through photon-graviton conversion, however, is negligible due to plasma effects.

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